Patrick Trahan

Formula Sheet

Mean

Variance

Standard deviation

Empirical Rule

For a distribution of measurements that is approximately normal (bell shaped), it follows that the interval with end points

contains approximately 68% of the measurements.

contains approximately 95% of the measurements.

contains almost all of the measurements.

Definition 2.3

The *sample space* associated with an experiment is the set consisting of all possible sample points. A sample space will be denoted by S.

Probability of A

Axiom 1:

Axiom 2:

Axiom 3: If form a sequence of pairwise mutually exclusive events in S, then

Sample-point Method

The following steps are used to find the probability of an event.:

1. Define the experiment and clearly determine how to describe one simple event.
2. List the simple events associated with the experiment and test each to make certain that it cannot be decomposed. This defines the sample space .
3. Assign reasonable probabilities to the sample points in , making certain that and .
4. Define the event of interest, A, as a specific collection of sample points. (A sample point is in A if A occurs when the sample point occurs when the sample point occurs. Test *all* sample points in to identify those in A.)
5. Find by summing the probabilities of the sample points in A.

Theorem 2.1

with elements and elements it is possible to form pairs containing one element from each group.

Permutation

The number of ways of ordering distinct objects taken at a time will be designated by the symbol

Theorem 2.2

Theorem 2.3

Combination

or

Theorem 2.4

Conditional Probability

where

Independent

Otherwise, its dependent

Multiplicative Law of Probability

Two events A and B is

If A and B are independent, then

Additive Law of Probability

Theorem 2.7

A summary of the steps used in the event-composition method follows:

1. Define the experiment.
2. visualize the nature of the sample points. Identify a few to clarify your thinking.
3. Write an equation expressing the event of interest -say, A – as a composition of two or more events, using unions, intersections, and/or complements. (Notice that this equates point sets.) Make certain that event A and the event implied by the composition represent the same set of sample points.
4. Apply the addictive and multiplicative laws of probability to the compositions obtained in step 3 to find .

Partition of S

1. for

is the partition of S

Theorem 2.8

Bayes’ Rule

Definition 2.12

A *random variable* is a real-valued function for which the domain is a sample space.

Definition 2.13

Let and represent the numbers of elements in the population and sample, respectively. If the sampling is conducted in such a way that each of the samples has an equal probability of being selected, the sampling is said to be random, and the result is said to be a *random sample*.

Definition 3.1

A random variable is said to be *discrete* if it can assume only a finite or countably infinite number of distinct values.

Definition 3.2

The probability that takes on the value , , is defined as the *sum of the probabilities of all sample points in* that are assigned that value . We will sometimes denote by .

Definition 3.3

The *probability distribution* for a discrete variable can be represented by a formula, a table, or a graph that provides for all .

Theorem 3.1

For any discrete probability distribution, the following must be true:

1. for all
2. , where the summation is over all values of with nonzero probability.

Expected Value

Variance

Definition 3.6

A *binomial experiment* possesses the following properties:

1. The experiment consists of a fixed number, , of identical trials.
2. Each trial results in one of two outcomes: success, , or failure, .
3. The probability of success on a single trial is equal to some value and remains the same from trial to trial. The probability of a failure is equal to .
4. The trials are independent.
5. The random variable of interest is , the number of successes observed during the trials.

Definition 3.7

A random variable is said to have a *binomial distribution* based on trials with success probability if and only if

, and

Theorem 3.7

Let be a binomial random variable based on trials and success probability . Then

and .

Definition 3.8

A random variable is said to have a *geometric probability distribution* if and only if

, .

Theorem 3.8

If is a random variable with a geometric distribution,

and

**Hypergeometric Probability Distribution**

Definition 3.10

A random variable is said to have a *hypergeometric probability distribution* if and only if

Where is an integer , subject to the restrictions and .

Theorem 3.10

If is a random variable with a hypergeometric distribution,

and

**Poisson Probability Distribution**

Definition 3.11

A random variable is said to have a *Poisson probability distribution* if and only if

Theorem 3.11

If is a random variable possessing a Poisson distribution with parameter , then

and

**Tchebysheff’s Theorem**

Theorem 3.14

**Tchebysheff’s Theorem** Let be a random variable with mean and finite variance . Then, for any constant ,

or

**The Probability Distribution for a Continuous Random Variable**

Definition 4.1

Let denote any random variable. The *distribution function* of , denoted by , is such that for .

Theorem 4.1

**Properties of a Distribution Function** If is a distribution function, then

1. .
2. is a nondecreasing function of . [If and are *any* values such that , then .]

Definition 4.2

A random variable with distribution function is said to be *continuous* if is continuous, for .

Definition 4.3

Let be the distribution function for a continuous random variable . Then , given by

Wherever the derivative exists, is called the *probability density function* for the random variable .

Theorem 4.2

**Properties of a Density Function** If is a density function for a continuous random variable, then

1. for all , .
2. .

Definition 4.4

Let denote any random variable. If , the pth *quantile* of , denoted by , is the smallest value such that . If is continuous, is the smallest value such that . Some prefer to call the 100th *percentile* of .

Theorem 4.3

If the random variable has density function and , then the probability that falls in the interval is

**Expected Values for Continuous Random Variables**

Definition 4.5

The expected value of a continuous random variable is

,

Provided that the integral exists.

Theorem 4.4

Let be a function of ; then the expected value of is given by

,

Provided that the integral exists.

Theorem 4.5

Let be a constant and let be functions of a continuous random variable . Then the following results hold:

1. .
2. .
3. .

**The Uniform Probability Distribution**

Definition 4.6

If , a random variable is said to have a continuous *uniform probability distribution* on the interval if and only if the density function of is

Definition 4.7

The constants that determine the specific form of a density function are called *parameters* of the density function.

Theorem 4.6

If and is a random variable uniformly distributed on the interval , then

and .

**Bivariate and Multivariate Probability Distributions**

Definition 5.1

Let and be discrete random variables. The *joint* (or bivariate) *probability function* for and is given by

Theorem 5.1

If and are discrete random variables with joint probability function , then

1. for all .
2. , where the sum is over all values that are assigned nonzero probabilities.

Definition 5.2

For any random variables and , the joint (bivariate) distribution function is

,

Definition 5.3

Let and be continuous random variables with joint distribution function . If there exists a nonnegative function , such that

,

For all , then and are said to be *jointly continuous random variables*. The function is called the *joint probability density function*.

Theorem 5.2

If and are random variables with joint distribution function , then

1. .
2. .
3. If and , then

.

Theorem 5.2

If and are jointly continuous random variables with a joint density function given by , then

1. for all .
2. .

**Marginal and Conditional Probability Distributions**

Definition 5.4

1. Let and be jointly discrete random variables with probability function . Then the *marginal probability functions* of and , respectively, are given by

and .

1. Let and be jointly continuous random variables with joint density function . Then the *marginal density functions* of and , respectively, are given by

and .

Definition 5.5

If and are jointly discrete variables with joint probability function and marginal probability functions and , respectively, then the *conditional discrete probability function* of given is

,

Provided that .

Definition 5.6

If and are jointly continuous random variables with joint density function , then the *conditional distribution function* of given is

.

Definition 5.7

Let and be jointly continuous random variables with joint density and marginal densities and , respectively. For any such that , the conditional density of given is given by

And, for any such that , the conditional density of given is given by

.

**Independent Random Variables**

Definition 5.8

Let have distribution function , have distribution function , and and have joint distribution function . Then and are said to be *independent* if and only if

For every pair of real numbers . If and are not independent, they are said to be *dependent*.

Theorem 5.4

If and are discrete random variables with joint probability function and marginal probability functions and , respectively, then and are independent if and only if

For all pairs of real numbers .

Theorem 5.4

If and are continuous random variables with joint density function and marginal density functions and , respectively, then and are independent if and only if

For all pairs of real numbers .

Theorem 5.5

Let and have a joint density that is positive if and only if and , for constants and ; and otherwise. Then and are independent random variables if and only if

Where is a nonnegative function of alone and is a nonnegative function of alone.